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A CONTRIBUTION TO THE PROBLEM OF CORRECT DESCRIPTION OF ELECTROMAGNETIC WAVES IN LAYERED MEDIA WITH MAGNETIC PROPERTIES

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To model electromagnetic phenomena in layered media when we have a spatial charge distribution and an electric double layer at the boundary of contact it is proposed that the equation of telegraphy for the vector of the electric field strength and 12 conditions at the boundary of contact which reflect the laws of conservation of charge and energy without separating explicitly the surface charge for the multidimensional case be employed. An example of solution of the problem of propagation of a plane monochromatic wave for two media with dissimilar electrophysical properties is given for the one-dimensional case.

Investigation of the interaction of electric and thermal fields with allowance for mass transfer and contact phenomena is a complicated and pressing problem of the theory and practice of various fields of natural science and technology. Two approaches are applicable to its solution.

We can consider in detail the action of an electric field on electric charges that exist independently or form part of the molecules or atoms of a medium. However the computations required in this case are cumbersome since it is necessary to take account of the action, on each charge, not only of the incident wave but also of the secondary waves from all the remaining charges [1, p. 302].

The other way of solving the problem relies on phenomenological electrodynamics whose propositions provide the basis for the investigations of the present work. Let us consider the interface S of two media with dissimilar electrophysical properties. The surface charges σ and the surface current $\mathbf{i}$ (vector lying in the tangential plane to the interface S) occur on the contact under the action of the external electric field. The vectors of the magnetic field strength $\mathbf{H}$ and of the magnetic induction $\mathbf{B}$ and the vectors of the electric field $\mathbf{E}$ and of the electric displacement $\mathbf{D}$ are finite and continuous on both sides of the interface but they can experience a discontinuity of the first kind at the phase boundary.

In considering the electric field interacting with a material medium, we use the Maxwell equations [1, p. 299]

$$\frac{\partial \mathbf{D}}{\partial t} + \mathbf{I}_q = \nabla \times \mathbf{H} , \quad \nabla \mathbf{D} = \rho ; \tag{1}$$

$$-\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \mathbf{E} , \quad \nabla \mathbf{B} = 0 . \tag{2}$$

At the interface S, the system of equations is supplemented with the conditions

$$D_{\mathrm{n}1} - D_{\mathrm{n}2} = \sigma , \tag{3}$$

$$E_{\tau 1} - E_{\tau 2} = 0 , \tag{4}$$

$$B_{\mathrm{n}1} - B_{\mathrm{n}2} = 0 , \tag{5}$$

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$$H_{\tau 1} - H_{\tau 2} = i_\tau . \tag{6}$$

We note that by the subscript τ one can mean any direction tangential to the discontinuity surface.

The surface charge σ is formed owing to the spontaneous redistribution of ions or electrons at the boundary of a layered medium for equalization of Fermi energy levels [2, p. 425]. An electric double layer results, and we have a spatial electric-charge distribution near the boundary of contact of dissimilar substances. The structure of the electric double layer is affected by nonstationary thermal and diffusion processes, which makes the problem of modeling of electric fields even more complicated.

Various reasons can be responsible for the charge distribution. For example, in the case of electrolyte–metal contact it is attributed to the ions going from the electrode into the solution and to the specific adsorption of ions of one sign on the electrode surface and to the orientation of polar molecules near the electrode surface [3, p. 39]. Other reasons are responsible for the structure of the electric double layer in contact of two solid semiconductors or of a dielectric and a semiconductor, and this structure has its own features [4, p. 490; 5].

We note that the structure of the electric double layer substantially affects electrokinetic phenomena, the rate of electrochemical processes, and the stability of colloidal systems.

For the above reasons the electric double layer leads to fundamental difficulties in modeling the electric fields in a layered medium. Construction of the equivalent circuits for taking into account the electric double layer through introduction of a surface capacitance [5] (that is found experimentally) is worthwhile only for the range of conditions under which it has been determined.

This work seeks to construct a physicomathematical model of interaction of nonstationary electric fields in a layered medium without separating explicitly the surface charge. Contacting media are considered to be homogeneous.

Having eliminated the magnetic field strength from system (1)–(2), we obtain the known equation for the electric field strength:

$$\frac{\varepsilon}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} + \mu_0 \frac{\partial \mathbf{I}_q}{\partial t} = \frac{1}{\mu}\nabla^2 \mathbf{E} . \tag{7}$$

In Cartesian coordinates, it will have the form

$$\begin{aligned}
&\frac{\varepsilon}{c^2}\frac{\partial^2 E_x}{\partial t^2} + \mu_0 \frac{\partial I_{qx}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right),\\
&\frac{\varepsilon}{c^2}\frac{\partial^2 E_y}{\partial t^2} + \mu_0 \frac{\partial I_{qy}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right),\\
&\frac{\varepsilon}{c^2}\frac{\partial^2 E_z}{\partial t^2} + \mu_0 \frac{\partial I_{qz}}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right).
\end{aligned} \tag{8}$$

At the interface, a relation holds that reflects the law of conservation of electric charge [6, p. 420]:

$$\operatorname{div} \mathbf{i} + I_{qx_1} - I_{qx_2} = -\frac{\partial \sigma}{\partial t} , \tag{9}$$

where $\mathbf{i} = i_y \cdot \mathbf{j} + i_z \cdot \mathbf{h}$ is the surface-current density [6, p. 180]; the coordinate x is directed along the normal to the boundary.

Conditions (3)–(6) will be written in a Cartesian coordinate system:

$$D_{x1} - D_{x2} = \sigma , \tag{10}$$

$$E_{y1} - E_{y2} = 0 , \tag{11}$$

$$E_{z1} - E_{z2} = 0 , \tag{12}$$

$$B_{x1} - B_{x2} = 0 , \tag{13}$$

$$H_{y1} - H_{y2} = i_y , \tag{14}$$

$$H_{z1} - H_{z2} = i_z . \tag{15}$$

By the density i_y and i_z of the surface current we mean the quantity of electricity traversing, per unit time, a unit length of the segment which is located perpendicularly to the direction of the current on the surface carrying it. The surface density of displacement currents is always equal to zero if $\partial \mathbf{D}/\partial t$ has a finite value [6, p. 344]; therefore, the surface current cannot cause the surface charge to change. The value of the surface current is low for common materials by virtue of the small cross section; however, it can turn out to be substantial for superconductors. In what follows, we will disregard the surface current.

Indeed, no surface electric currents exist under actual conditions when the electrical conductivities of media are finite. The presence of them would imply that a current of finite value traverses an infinitely small cross section of the conductor. In this case an infinitely high power is consumed in the volume of finite dimensions. The possibility of disregarding surface currents for media with a finite conductivity is substantiated in greater detail in [7, p. 227].

Differentiating expression (10) with respect to time and taking into account relation (9), at the boundary of the media we obtain the condition of equality of the normal components of the total current:

$$I_{qx1} + \frac{\partial D_{x1}}{\partial t} = I_{qx2} + \frac{\partial D_{x2}}{\partial t} . \tag{16}$$

This condition enables us to eliminate the surface density of the charge of the electric double layer from consideration. Let $\mathbf{D} = \varepsilon\varepsilon_0\mathbf{E}$ and $\mathbf{I}_q = \lambda\mathbf{E}$. We introduce the notation $[f]\big|_{x=\xi} = f_1\big|_{x=\xi+0} - f_2\big|_{x=\xi-0}$ for the arbitrary function f. Then expression (16) will take the form

$$\left[\lambda E_x + \varepsilon\varepsilon_0 \frac{\partial E_x}{\partial t}\right]\Bigg|_{x=\xi} = 0 . \tag{16a}$$

Next, acting on the left-hand and right-hand sides of Eq. (1) by the operator div and taking into account that divrot $\mathbf{H} = 0$, we obtain

$$\frac{\partial}{\partial t}(\mathrm{div}\,\mathbf{D}) + \mathrm{div}\,(\mathbf{I}_q) = 0 .$$

Consequently, the relations reflecting the law of conservation of electric charge hold at the boundary of media 1 and 2 in the Cartesian coordinate system:

$$\varepsilon_1\varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_{1x}}{\partial x} + \frac{\partial E_{1y}}{\partial y} + \frac{\partial E_{1z}}{\partial z}\right) + \lambda_1\left(\frac{\partial E_{1x}}{\partial x} + \frac{\partial E_{1y}}{\partial y} + \frac{\partial E_{1z}}{\partial z}\right) = 0 , \tag{17}$$

$$\varepsilon_2\varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial E_{2x}}{\partial x} + \frac{\partial E_{2y}}{\partial y} + \frac{\partial E_{2z}}{\partial z}\right) + \lambda_2\left(\frac{\partial E_{2x}}{\partial x} + \frac{\partial E_{2y}}{\partial y} + \frac{\partial E_{2z}}{\partial z}\right) = 0 . \tag{18}$$

We believe that E_x is a continuous function of y and z at the boundary of the media. Then, upon differentiating (16a) with respect to y and z, we have

$$\left[\lambda \frac{\partial E_x}{\partial y} + \varepsilon\varepsilon_0 \frac{\partial^2 E_x}{\partial y \partial t}\right]\Bigg|_{x=\xi} = 0\,, \tag{19}$$

$$\left[\lambda \frac{\partial E_x}{\partial z} + \varepsilon\varepsilon_0 \frac{\partial^2 E_x}{\partial z \partial t}\right]\Bigg|_{x=\xi} = 0\,. \tag{20}$$

Let us differentiate conditions (13)–(15) with respect to time for the magnetic induction and the magnetic field strength. Having set $\mathbf{B} = \mu\mu_0\mathbf{H}$, we obtain (without taking account of the influence of surface currents)

$$\left[\frac{\partial B_x}{\partial t}\right]\Bigg|_{x=\xi} = 0\,, \quad \left[\frac{1}{\mu\mu_0}\frac{\partial B_y}{\partial t}\right]\Bigg|_{x=\xi} = 0\,, \quad \left[\frac{1}{\mu\mu_0}\frac{\partial B_z}{\partial t}\right]\Bigg|_{x=\xi} = 0\,. \tag{21}$$

Having taken (2) into account, we express (21) in terms of the projections of the rotation of the electric field:

$$[\mathrm{rot}_{x1}\,\mathbf{E}]_{x=\xi} = 0 \quad \text{or} \quad \left[\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right]\Bigg|_{x=\xi} = 0\,, \tag{22}$$

$$\left[\frac{1}{\mu\mu_0}\mathrm{rot}_y\,\mathbf{E}\right]\Bigg|_{x=\xi} = 0 \quad \text{or} \quad \left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x}\right)\right]\Bigg|_{x=\xi} = 0\,, \tag{23}$$

$$\left[\frac{1}{\mu\mu_0}\mathrm{rot}_z\,\mathbf{E}\right]\Bigg|_{x=\xi} = 0 \quad \text{or} \quad \left[\frac{1}{\mu\mu_0}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right]\Bigg|_{x=\xi} = 0\,. \tag{24}$$

Here (22) is the normal projection of the rotation, (23) is the tangential projection of the rotation of the electric field in y, and (24) is the projection of the rotation in z.

Assuming that E_y and E_z are continuous differentiable functions of the coordinates y and z, from conditions (11) and (12) we obtain

$$\left[\frac{\partial E_y}{\partial y}\right]\Bigg|_{x=\xi} = 0\,, \quad \left[\frac{\partial E_y}{\partial z}\right]\Bigg|_{x=\xi} = 0\,; \tag{25}$$

$$\left[\frac{\partial E_z}{\partial y}\right]\Bigg|_{x=\xi} = 0\,, \quad \left[\frac{\partial E_z}{\partial z}\right]\Bigg|_{x=\xi} = 0\,. \tag{26}$$

To solve the complete system of equations (8) in the general case we must determine 12 boundary conditions at the boundary of the adjacent media:

a) the functions E_x, E_y, and E_z are determined by conditions (11), (12), and (16a);

b) the derivatives $\partial E_x/\partial x$, $\partial E_y/\partial y$, and $\partial E_z/\partial z$ are found from conditions (17) and (18) as direct consequences of the laws of conservation of charge;

c) the quantities $\partial E_x/\partial y$ and $\partial E_y/\partial z$ are calculated from relations (19) and (20) with allowance for the continuity of the normal component of the total current (16a) in the coordinates x, y, and z at the boundary;

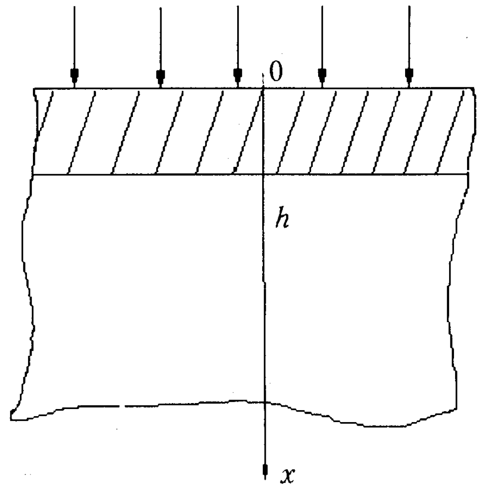

Fig. 1. Scheme of contact of the plate with the semiinfinite medium.

d) the values of $\partial E_y/\partial y$, $\partial E_y/\partial z$, $\partial E_z/\partial y$, and $\partial E_z/\partial z$ are determined by conditions (25) and (26) as a consequence of the continuity of the tangential components of the electric field in y and z;

e) the derivatives $\partial E_x/\partial z$, $\partial E_x/\partial y$, $\partial E_y/\partial x$, and $\partial E_z/\partial y$ are found from conditions (23) and (24) as a consequence of the equality of the tangential components of the rotation of the electric field in y and z.

We note that condition (16a) has been employed in [8] in numerical modeling of pulsed electrochemical processes. For the normal component of the rotation of the electric field relation (22) is a linear combination of conditions (25) and (26); therefore, $\text{rot}_x\, \mathbf{E} = 0$. In what follows, we do not employ this condition. Equations (17) and (18) determining the values of $\partial E_x/\partial x$, $\partial E_y/\partial y$, and $\partial E_z/\partial z$ in the adjacent media 1 and 2 are actually a "single" condition at the boundary. The special properties of the expression of the general law of conservation of electric charge at the boundary imply that the components $\partial E_y/\partial y$ and $\partial E_z/\partial z$ are determined from conditions (25) and (26) that are derived from the equality and continuity of the tangential components E_y and E_z at the boundary of the adjacent media.

Example. Electromagnetic phenomena occurring in the case of incidence of plane electromagnetic waves on the interfaces of dissimilar media play an important role in engineering, since all the actual devices are bounded by the surfaces and are nonuniform in space [9, p. 87]. Let us dwell on the simplest case of propagation of a plane monochromatic wave through the boundary of two media for the one-dimensional case where the amplitude of E_y depends only on x.

We consider the contact of an infinite plate of thickness h, which has constant electrophysical properties ε_1, μ_1, and λ_1, with a semiinfinite medium having properties ε_2, μ_2, and λ_2. On the surface of the plate $x = 0$, there is a source of a periodic boundary regime with a cyclic frequency ω (Fig. 1). In this case, the equation for the electric field strength (8) in each of the media has the form

$$\frac{\varepsilon}{c^2}\frac{\partial^2 E_y}{\partial t^2} + \mu_0\,\lambda\,\frac{\partial E_y}{\partial t} = \frac{1}{\mu}\left(\frac{\partial^2 E_y}{\partial x^2}\right); \tag{27}$$

here,

$$x = 0: \quad E_y\,(0, t) = A \exp\,(i\omega t) \quad \text{and} \quad x = \infty: \quad E_y\,(\infty, t) = 0\,. \tag{28}$$

At the boundary of contact $x = h$, the following two conditions must be satisfied:

(1) equality of the tangential components

$$E_{y1} = E_{y2}\,, \tag{29}$$

(2) equality of the tangential projections of the rotation of the electric field

$$\frac{1}{\mu_1}\frac{\partial E_{y1}}{\partial x}=\frac{1}{\mu_2}\frac{\partial E_{y2}}{\partial x}\,. \tag{30}$$

Here it is taken that $\partial E_{x1}/\partial y = 0$ and $\partial E_{x2}/\partial y = 0$, since $E_z = 0$.

Let $E_y = \tilde{E}_y(x)\exp(i\omega t)$. We introduce the notation $E_y(x)\equiv E$; then Eq. (27) will be represented as

$$\frac{\partial^2 E}{\partial x^2}-k^2E=0\,, \tag{31}$$

$$k^2=-\frac{\mu\omega^2\varepsilon}{c}+i\omega\lambda\,\mu_0=-a+ib\,. \tag{32}$$

The solution of (27)–(30) is sought in the form of a superposition of a traveling wave and a reflected wave:

$$E_1=C_1\exp(k_1x)+D_1\exp(-k_1x)\quad\text{(in the plate)}\,, \tag{33}$$

$$E_2=C_2\exp(k_2x)+D_2\exp(-k_2x)\quad\text{(in the semiinfinite medium)}\,, \tag{34}$$

The quantities E_1 and E_2 satisfy Eq. (31). From boundary conditions (28) we have

$$C_1+D_1=A\,, \tag{35}$$

$$C_2=0\,, \tag{36}$$

since $E\to 0$ when $x\to\infty$.

Conditions (29) and (30) on the contact can be reduced to the system of algebraic equations

$$C_1\exp(k_1h)+D_1\exp(-k_1h)=D_2\exp(-k_2h)\,, \tag{37}$$

$$\mu_1^{-1}(C_1\exp(k_1h)-D_1\exp(-k_1h))=-\mu_2^{-1}k_2D_2\exp(-k_2h)\,. \tag{38}$$

Upon transformations, the expression for the constants has the final form

$$C_1=\frac{A\exp(-k_1h)\,(k_1\,\mu_1^{-1}-\mu_2^{-1}k_2\exp(-k_2h))}{k_1\,\mu_1^{-1}(\exp(k_1h)+\exp(-k_1h))+\mu_2^{-1}k_2\exp(-k_2h)\,(\exp(k_1h)-\exp(-k_1h))}\,, \tag{39}$$

$$D_1=A-C_1\,,\quad C_2=0\,, \tag{40}$$

$$D_2=A\exp(k_1h)\left[\frac{k_2\,\mu_2^{-1}-\mu_2^{-1}k_2\exp(-k_1h)}{k_1\,\mu_1^{-1}(\exp(k_1h)+\exp(-k_1h))+\mu_2^{-1}k_2\exp(-k_2h)\,(\exp(k_1h)-\exp(-k_1h))}+\right.$$
$$\left.+1-\frac{\exp(-2k_1h)\,(k_1\,\mu_1^{-1}-\mu_2^{-1}k_2\exp(-k_1h))}{k_1\,\mu_1^{-1}(\exp(k_1h)+\exp(-k_1h))+\mu_2^{-1}k_2\exp(-k_2h)\,(\exp(k_1h)-\exp(-k_1h))}\right]. \tag{41}$$

Condition (30) at the boundary of contact is new and it enables us to take into account the influence of the magnetic properties of a substance.

In specifying the boundary conditions, the widest acceptance has been gained at present by the approximate impedance conditions of Leontovich and Shchukin [9, p. 87]; these condition relate the tangential components of the electromagnetic field at the interface of two media through introduction of the impedance z, $E_\tau = zH_\tau$. It is emphasized in [9, p. 97] that, generally speaking, z is a function of the process and it is not a constant; furthermore, z can have a tensor integro-differential operator form. In [11, 12], the thickness of the transition layer in which the change in electrophysical properties is a continuous function of the coordinate is artificially introduced to model layered media. In this approach, solution of the problem is mainly determined by the form of a smoothing polynomial and the thickness of the transition layer.

In computing $k_1 = \pm\sqrt{-a_1 + ib_1} = u_1 + iv_1$ and $k_2 = \pm\sqrt{-a_2 + ib_2} = u_2 + iv_2$, it must be borne in mind that the sign of b determines selection of the signs of u and v according to the relation $2uv = b$. When $b > 0$, u and v have opposite signs; here

$$u^2 = \frac{1}{2}\left(a^2 + \sqrt{a^2 + b^2}\right),$$

$$v^2 = \frac{1}{2}\left(-a^2 + \sqrt{a^2 + b^2}\right).$$

For media with relaxation we represent expression (22) in a more complex form [10]:

$$k^2 = -\frac{\omega^2 \varepsilon\varepsilon_0 \mu_0 \mu}{1 + \omega\tau_r^2} + i\left(\frac{\mu\mu_0\varepsilon\varepsilon_0\tau_p\omega^3}{1 + \omega^2\tau_r^2} + \lambda\omega\right). \quad (42)$$

In this relation, account is taken of the delay of dipoles and domains in the external electromagnetic field which is attributed not only to the electrical phenomena of relaxation but also to the magnetic ones. In deriving (42), we employed material equations of the form [10]:

$$D(t) + \tau_r \frac{dD(t)}{dt} = \varepsilon\varepsilon_0 E(t),$$

$$B(t) + \tau_r \frac{dB(t)}{dt} = \mu\mu_0 H(t).$$

It was assumed that the relaxation times τ_r of the electric and magnetic fields coincide. This is not necessarily so for magnetic materials. In what follows, we are planning to take account of the general case where the relaxation times of the field differ.

The numerical investigation of propagation of a plane monochromatic wave through the boundary of two media carried out for the one-dimensional case has shown that the character of change of the electric-field amplitude along the x axis is determined by the difference of the contacting media in electrophysical properties.

The dimensionless amplitude of the electric field as a function of the penetration depth of the electromagnetic wave ($\omega = 10^9$) in a two-layer medium with allowance for the relaxation of the electric and magnetic fields is illustrated by the curves presented in Fig. 2. Medium 1 is a plate with a thickness of 0.1 m, and medium 2 is a semiinfinite space. The electrophysical properties of the first medium were considered to be constant and equal to $\lambda_1 = 0$, $\varepsilon_1 = 15$, $\mu_1 = 1$, and $\tau_{r1} = 0$.

Figure 2A shows the influence of the electrical conductivity of both media on the amplitude of the electric field (the magnetic properties of the media were also varied). Figure 2B, conversely, plots the amplitude of the electric field versus the relative permeability.

Analysis of the behavior of the curves presented in these figures shows that if the permeabilities of the adjacent media are equal, the curves characterizing the dimensionless amplitude of the electric field as a function of the

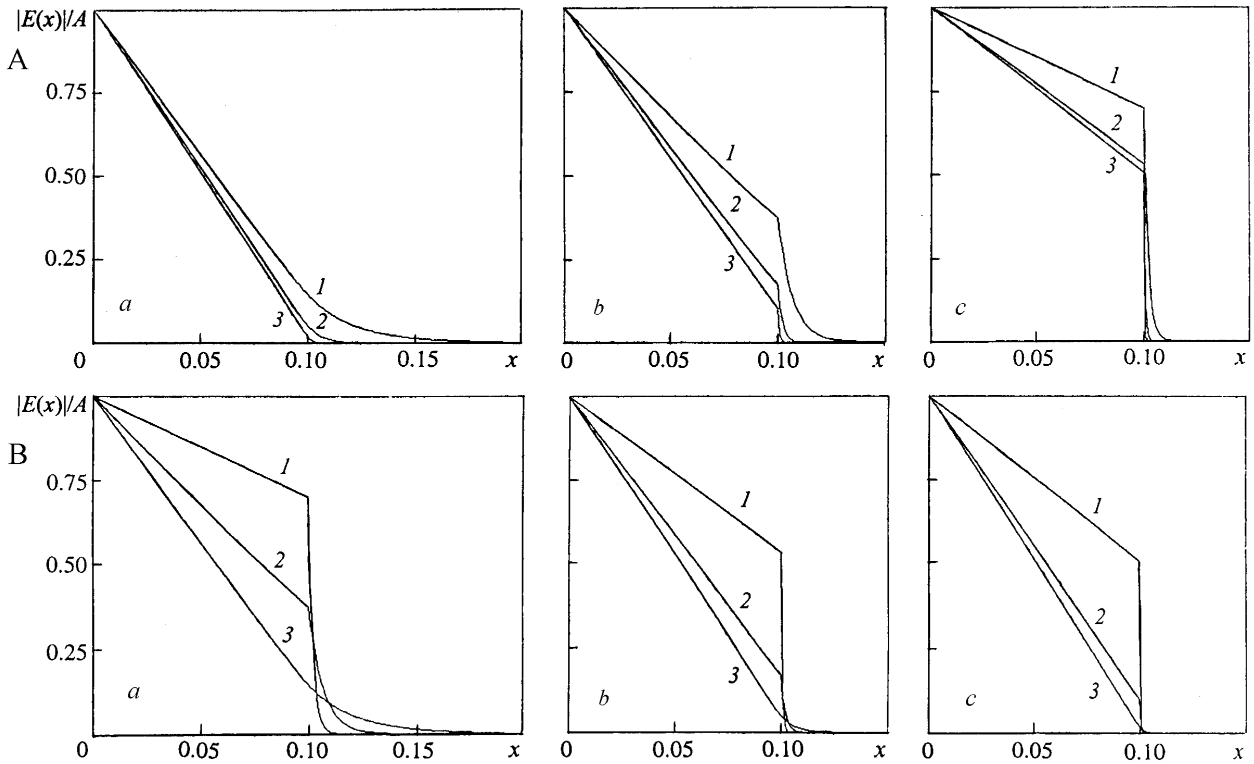

Fig. 2. Dimensionless amplitude of the electric field vs. penetration depth of the electromagnetic wave ($\omega = 10^9$) in a two-layer medium (ε_1 = 15, τ_{r1} = 0, μ_1 = 1 and λ_1 = 0; ε_2 = 20 and τ_{r2} = 10^{-10}): A) λ_2 = 10 (1), 100 (2), and 1000 (3); μ_2 = 1 (a); μ_2 = 10 (b); μ_2 = 100 (c); B) μ_2 = 10 (1), 100 (2), and 1000 (3); λ_2 = 10 (a), 100 (b), and 1000 (c).

penetration depth of the electromagnetic wave represent continuous smooth lines (Fig. 2A, a). In the case where $\mu_1 \neq \mu_2$ such curves have bends (Fig. 2A, b and c and Fig. 2B).

We note that the relaxation times can also exert a substantial influence on the propagation of the electric wave (the data are not given).

Thus, to model electromagnetic phenomena in layered media when we have a spatial distribution of electric charges and an electric double layer at the boundary of contact it is proposed that the equation of telegraphy (for the vector of the electric field strength) and the law of conservation of electric charge be employed. In this case it is unnecessary to specify the surface charge, the surface capacitance, or the Leontovich–Shchukin impedance condition, which are not only characteristics of the properties of the surface but also a function of the process. At the boundary of the adjacent media, we derive and substantiate the following conditions: equality of the normal components of the total current, equality of the tangential projections of the rotation of the electric field $\left[\frac{1}{\mu\mu_0}\,\mathrm{rot}_\tau \mathbf{E}\right]\Big|_{x=\xi} = 0$, the law of conservation of electric charge, equality of the tangential components of the electric field and their derivatives in the tangential direction, and equality of the derivatives of the normal components of the electric field in the direction tangential to the interface of the adjacent media.

We have given the example of solution of the problem of propagation of a plane monochromatic wave for two media with dissimilar electrophysical properties for the one-dimensional case. We have investigated numerically the influence of the magnetic properties of a medium on the propagation of an electromagnetic wave with allowance for relaxation.

The calculation results are in qualitative agreement with the experimental data.

The approach proposed enables one to construct for the first time the consistent physico-mathematical model of propagation of electromagnetic waves in a layered medium and to compute the charge of an electric double layer based on the equations of macroscopic phenomenological electrodynamics.

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NOTATION

B, magnetic induction; **D**, electric displacement, C/m^2; **E**, electric field strength, V/m; **H**, magnetic field strength, A/m; $\mathbf{I}_q$, charge-flux density, C/(m^2·sec); **i**, surface current, A/m; j and h, densities of the surface currents in the coordinates y and z respectively; q, charge; ρ, charge density, C/m^3; σ, surface charge density, C/m^2, ε, relative permittivity; ε_0, electric constant; λ, specific electrical conductivity; μ, relative permeability; μ_0, magnetic constant; c, electrodynamic constant; μ_r, permeability; τ_r, relaxation time of dipoles or domains; ω, cyclic frequency of the wave; t, time; x, y, z, Cartesian coordinates; a and b, real and imaginary parts of the complex number. Subscripts: 1 and 2, first and second medium, n and τ, normal and tangential directions to the discontinuity surface; x, normal component of the vector; y and z, tangential components of the vector at the boundary of the adjacent media; r, relaxation.

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